detector used was merely a level-indicating device with readings taken on a rotary vane attenuator to obtain the same power level with or without the harmonic suppressor in the line. For second harmonics, sweep techniques were used to verify suppressor performance. In effect, sweep eliminates the possibility of missing narrow ranges where performance may not be up to specification. Figs. 8 and 9 show reproductions of oscilloscope traces obtained in this test. Figs. 10 and 11 show a table of device characteristics for higher frequency ranges where sweep equipment was not available.

## Conclusions

Two different harmonic suppressors finalizing the above design principle have been developed, one operating in $C$ band, and the other in $X$ band. Up to this date these have only been tested at high CW power levels and no peak power breakdown data exists. Figs. 10 and 11 also show a summary of important device parameters such as size and weight, attenuation per unit length, and a figure of merit which has been defined as attenuation for the second harmonic divided by attenuation for the fundamental. This figure of merit cannot be approached by reactive filters or by ferrite devices.

# A Plasma-Column Band-Pass Microwave Filter*

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*Summary*—**A tunable band-pass filter using the dipole resonance of a plasma column has been investigated. The center frequency of the pass band can be electronically tuned over a large portion of a waveguide band. For the prototype investigated at $S$ band, the insertion loss at the center frequency was less than 2 db, with isolation for frequencies outside the pass band on the order of 12 db. A typical 3-db bandwidth of this prototype was 150 Mc. It is expected that this figure can be improved by choice of better discharges than the positive column of the mercury discharge used here.**

**An analysis of the external $Q$'s for the input and output coupling is presented. From these calculations, it is possible to determine approximately the various coupling parameters that produce a given degree of overcoupling.**

## I. Introduction

BAND-PASS filters that can be electronically tuned over a wide band are desirable components in some present-day electronic systems. Several schemes have been proposed and investigated for achieving such filters. In most cases these have involved resonant cavities which are perturbed and tuned by a variable impedance element, such as a voltage variable capacity diode,[1] ferrite,[2–4] or ferroelectric.[5] A much wider tunable bandwidth can be obtained, however, by use of a material which itself exhibits a resonance effect. A particularly successful example of this is the magnetically tunable yttrium iron garnet filter investigated by Carter,[6] in which the spin resonance is used as a microwave resonator. The resonance exhibited by the plasma column can be used to make an electronically tunable filter in a similar way. This paper discusses the mechanism of such a filter and experimental results of a prototype filter constructed at $S$ band. While results for this prototype certainly do not rival those that have been achieved with the yttrium iron garnet filter, it is possible that the principle of the plasma filter may also find applications.

The plasma resonance used in our filter is that previously investigated by Tonks[7] and more recently by others, in particular by Dattner.[8] The simplified physical picture of the resonance is that of a cylindrical electron cloud oscillating about a stationary cylindrical ion cloud. This resonance can be treated as that of a microwave resonator, whose resonant frequency is a function of the plasma density.

The resonance of a plasma column is easily excited by passing the column through a $TE_{10}$ rectangular waveguide, such that the column axis is perpendicular to the incident electric field and to the direction of propagation. By positioning a pick-up probe so that it is only excited when the column is in resonance, a band-

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[1] A. Uhlir, Jr., "The potential of semiconductor diodes in high-frequency communications," Proc. IRE, vol. 46, pp. 1099–1115; June, 1958.

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[4] C. E. Nelson, "Ferrite-tunable microwave cavities and the introduction of a new reflectionless, tunable microwave filter," Proc. IRE, vol. 44, pp. 1449–1455; October, 1956.

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[6] P. S. Carter, Jr., "Magnetically-tunable microwave filters using single-crystal yttrium-iron-garnet resonators," IRE Trans. on Microwave Theory and Techniques, vol. MTT-9, pp. 252–260; May, 1961.

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[8] A. Dattner, "The plasma resonator," *Ericcson Technics* (Stockholm), vol. 13, pp. 310–350; 1957.

pass filter is made possible. By varying the discharge supply voltage and, hence, the plasma density, the resonant frequency of the column and, thus, the center of the pass band of the filter are electronically tuned.

An analysis of input and output coupling, given below, shows that the external $Q$ ($Q_e$) can be varied over a wide range by appropriate plasma and probe positioning. It is found that the values of $Q_e$ possible can be made several times smaller than the unloaded $Q$ ($Q_0$) of a typical plasma column, so that a low insertion-loss filter is possible.

## II. Principle of Operation

Several workers[7-10] have observed that a plasma column as shown in Fig. 1 will exhibit a resonance. The resonant frequencies of the column can be found from a static analysis under the condition that the dimensions are much less than a wavelength.

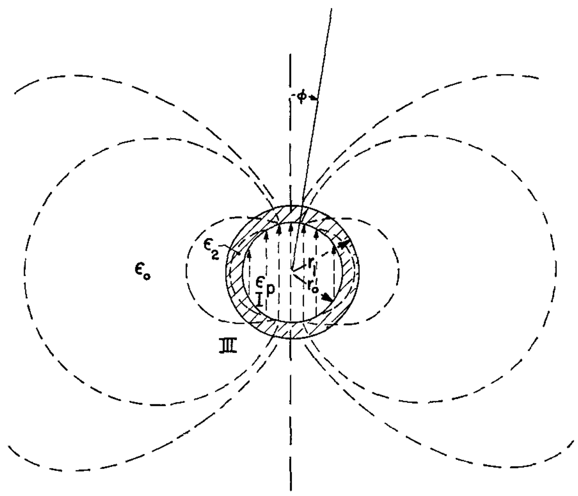


Fig. 1—Cross section of plasma column and near fields of the dominant dipole mode.

The problem can be handled[11,12] by solving Laplace's equation in the three regions (plasma, discharge tube, and free space) and matching tangential $\overline{E}$ and normal $\overline{D}$ across the interfaces. The relative dielectric constant of the plasma is assumed to be

$$\epsilon_p = \left[1 - \left(\frac{\omega_p}{\omega}\right)^2\right] \tag{1}$$

where

$\omega$ = signal frequency

$\omega_p$ = plasma frequency

$$= 5.6 \times 10^4 \times \left(\frac{\text{no. of electrons}}{cc}\right)^{1/2}.$$

For the mode with one angular variation and no axial variation the resonant frequency is found to be[9]

$$\omega_R = \omega_p[1 + K_{eff}]^{-1/2} \tag{2}$$

where

$$K_{eff} = \epsilon_2 \frac{\epsilon_2\left[\left(\frac{r_1}{r_0}\right)^2 - 1\right] + \left[\left(\frac{r_1}{r_0}\right)^2 + 1\right]}{\epsilon_2\left[\left(\frac{r_1}{r_0}\right)^2 + 1\right] + \left[\left(\frac{r_1}{r_0}\right)^2 - 1\right]}. \tag{3}$$

Here

$\epsilon_2$ = relative dielectric constant of the discharge tube,
$r_1$ = outer diameter of the discharge tube,
$r_0$ = inner diameter of the discharge tube.

Higher-order modes with larger angular and axial mode numbers are also possible. The higher-order angular modes lie on the high plasma-density side of the dominant mode. Since they are not as easily excited as the mode with one angular variation, no use is made of them in the filter.

An axial variation of the form sin $(m\pi x/a)$ is appropriate for the case of the column inserted across a rectangular waveguide of width $a$. For this finite length column we find, therefore, that the resonant frequencies are now given by

$$\omega_r = \omega_p[1 + K'_{eff}]^{-1/2} \tag{4}$$

where

$$K'_{eff} = \frac{\left(\frac{m\pi r_0}{a}\right) K_0\left(\frac{m\pi r_0}{a}\right) + K_1\left(\frac{m\pi r_0}{a}\right)}{\left(\frac{m\pi r_0}{a}\right) I_0\left(\frac{m\pi r_0}{a}\right) - I_1\left(\frac{m\pi r_0}{a}\right)} \times \frac{I_1\left(\frac{m\pi r_0}{a}\right)}{K_1\left(\frac{m\pi r_0}{a}\right)}. \tag{5}$$

$K_{0,1}\left(\frac{m\pi r_0}{a}\right)$ = modified Bessel function of the second kind.

$I_{0,1}\left(\frac{m\pi r_0}{a}\right)$ = modified Bessel function of the first kind.

The modes with values of $m > 1$ always appear on the high-density side of the dominant mode given by (2).

[9] G. D. Boyd, "Experiments on the Interaction of a Modulated Electron Beam with a Plasma," Calif. Inst. of Technology, Pasadena, Calif., Tech. Rept. No. 11, NONR 220 (13); May, 1959.

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[12] T. R. Kaiser and R. L. Closs, "Theory of radio reflections from meteor trails; I," *Phil. Mag.*, vol. 43, pp. 1–32; January, 1952.

To derive (4) and (5), we neglected the dielectric of the discharge tube for simplicity. It is found that for the lower values of $m$ and for $a/r_0 \simeq 20$, which is typical, the resonant frequencies given by (4) coalesce to that given by (2). Therefore, for low values of $m$, the resonant frequency of the dominant mode of a typical plasma column across a waveguide is given by (2) to within the accuracy of the static approximation.

Several workers[8–10,13] have reported the appearance of resonances on the low-density side of the dominant mode. These do not correspond to the higher-order modes discussed here. The origin of these modes has not been definitely determined to the satisfaction of all the investigators of this problem.

In the static approximation, the near fields of the dominant mode are entirely electric and for large values of $a/r_1$ are entirely in the transverse plane. At resonance, the energy is alternately stored in the electric field and in the kinetic energy of the plasma electrons. For the column inserted through a rectangular waveguide of width $a$, these near fields are approximated by the near fields of a column of length $a$ in a free space, provided the column is not in the immediate proximity of the top or bottom wall. These fields are then:

In the plasma:

$$E_1 = [\hat{\imath}_r \cos\phi - \hat{\imath}_\phi \sin\phi]\left[A_1 \sin\frac{\pi x}{a}\right].$$

In the dielectric tube:

$$E_2 = \{\hat{\imath}_r[A_2 - B_2 r^{-2}]\cos\phi + \hat{\imath}_\phi[-A_2 - Br^{-2}]\sin\phi\} \cdot \left\{\sin\frac{\pi x}{a}\right\}.$$

In the region external to the tube:

$$E_3 = [\hat{\imath}_r \cos\phi + \hat{\imath}_\phi \sin\phi]\left[-A_3 r^{-2}\sin\frac{\pi x}{a}\right]. \qquad (6)$$

Here $A_1$, $A_2$, $A_3$, and $B_2$ are constants determined by the power level; $r$, $\phi$, $x$ are cylindrical coordinates; $\hat{\imath}_r$, $\hat{\imath}_\phi$ are unit vectors. In addition to determining the condition for resonance, the matching of tangential $E$ and normal $D$ also relates the four constants, as given by (7); so that the constant $A_1$ completely determines the amplitude of the resonance oscillations. Thus,

$$\begin{aligned} A_2 &= A_1[1 - \epsilon_2][1 - \epsilon_2 - r_1^2 r_0^{-2}(1+\epsilon_2)]^{-1} \\ B_2 &= A_1[1 + \epsilon_2][-r_1^2][1 - \epsilon_2 - r_1^2 r_0^{-2}(1+\epsilon_2)]^{-1} \\ A_3 &= A_1[-2r_1^2\epsilon_2][1 - \epsilon_2 - r_1^2 r_0^{-2}(1+\epsilon_2)]^{-1}. \qquad (7) \end{aligned}$$

It should be pointed out that losses within the plasma have been neglected in this analysis. These losses are due to collisions of the oscillating electrons with neutrals, ions, the container walls, and each other. These processes are often summarized by an effective collision frequency, $\nu_c$.[14] In general, the resonances will only exist, and the plasma filter will only be possible, for the region in which $\omega \gg \nu_c$. The losses in the dielectric tube can also be of importance, of course. These can usually be minimized by choice of a low loss dielectric.

In the static approximation of lossless plasma resonance, there is no external applied electric field, yet large coherent oscillations of the plasma electrons occur. In the actual resonance, therefore, such oscillations can be produced by the application of a small external electric field. However, coherent oscillations of electrons correspond to an alternating current, which radiates electromagnetic energy. The resonant plasma column across the waveguide behaves, accordingly, like a line of electric dipoles of high $Q$, which radiate or scatter electromagnetic energy only in the vicinity of their resonant frequency. This behavior makes the plasma column immediately into a band-elimination filter—a property demonstrated indirectly in the work of Dattner.

To use the plasma column as a band-pass filter at least two alternatives exist. The first of these, which is the obvious extension of the band-elimination filter, is shown in Fig. 2. Here the section of waveguide which contains the plasma column is fed through a circulator and is terminated in a matched load. When the plasma column is not at resonance, it is virtually inactive. Almost all of the energy supplied to its waveguide section passes through it and is, therefore, absorbed by the matched load. When the plasma current is set to the value that produces resonance at the frequency to be passed, however, the plasma column behaves as a strong electric dipole and reflects appreciably all of the incident power. When this reflected power passes through a circulator again, it is now led to the output.

This behavior has been demonstrated in the laboratory and is shown in the CRO display of Fig. 3. Here the input frequency was kept constant, while the plasma current was varied. It is seen that only at and near resonance any power is transmitted to the load. (Although the actual experiment that produced this display used a 20-db directional coupler for viewing reflected power, a circulator would produce the same result, but without the loss of 20 db.)

While this particular method of producing a band-pass filter is successful, it has the disadvantage of requiring a circulator or directional coupler. The need for either is eliminated, however, by the method of selective coupling of Fig. 4. Here a pickup probe is placed near the plasma column parallel to the waveguide axis.

[13] F. Boley, "Scattering of microwave radiation by a plasma column," *Nature*, vol. 182, p. 790; 1958.

[14] L. Goldstein, "Advances in Electronics and Electron Physics," Academic Press, Inc., New York, N. Y., vol. VII, pp. 399–503; 1955.

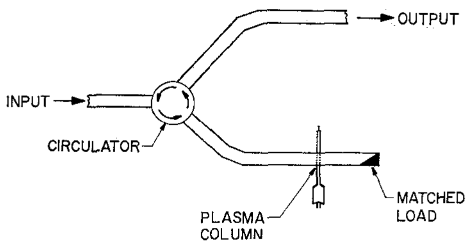


Fig. 2—Band-pass filter, using plasma column and circulator.

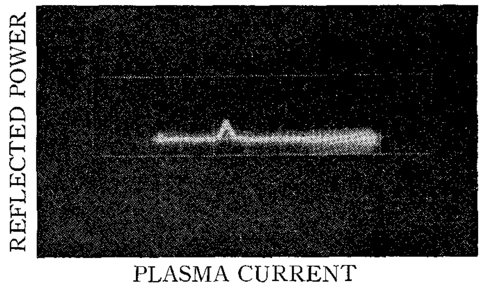


Fig. 3—Power reflected from plasma column at resonance.

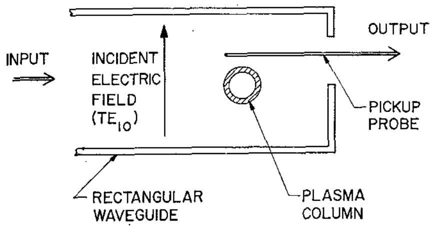


Fig. 4—Selective input and output coupling to column for filter.

Since the incident electric field is polarized perpendicular to this probe, it will not induce any currents in the probe in the absence of plasma resonance, and, therefore, no power will be coupled to the output. At the correct plasma density for resonance, however, the electric field will excite the resonance, which will generate the fields given by (6). These, of course, do have a component parallel to the probe and will, therefore, induce currents in the probe and the line feeding the output load. Again, by changing the plasma current and, thus, the plasma density, the center of the pass band of this filter can now be electronically altered. It is this embodiment of the plasma filter which is discussed in more detail below.

## III. Analysis of Coupling

An equivalent circuit for the plasma filter of Fig. 4, as viewed to the right in the plane of the waveguide at which the tube is located, is shown in Fig. 5(a). Here the two ideal transformers shown account for the degrees of coupling between the waveguide and plasma tube, and between the plasma tube and the output circuit. The shorted waveguide section through which the output coupling probe passes has an equivalent impedance of $jZ_0 \tan 2\pi c/\lambda_g$, where $Z_0$ is the waveguide impedance. We will initially assume that this section is of the order of $\lambda_g/4$ in the band over which the filter is to be tuned,

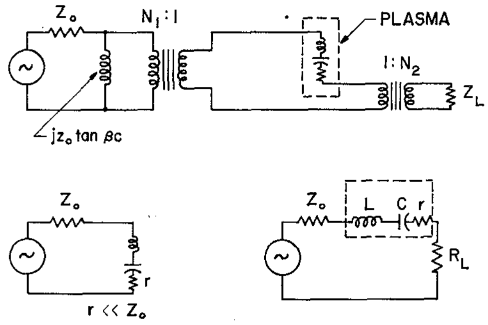


Fig. 5—Equivalent circuits for plasma filter.

so that its equivalent reactance is high and it can be neglected. The transformation ratios $N_1$ and $N_2$ are so chosen that very tight coupling corresponds to values of $N_1$ and $N_2$ smaller than unity. This is easily seen in the example of the plasma column across the guide, but with the coupling probe retracted. Here we have $N_1 \ll 1$, $N_2 \gg 1$ resulting in the approximate equivalent circuit of Fig. 5(b), in which the plasma behaves as a short across the guide as actually experienced.

To analyze the operation of the filter, we draw the equivalent circuit after the proper impedance conversions due to $N_1$ and $N_2$ have been performed. The result is the simple circuit of Fig. 5(c). We will now evaluate the performance of this circuit in terms of the behavior of the plasma column and the physical parameters.

The effective-percentage bandwidth of the filter is determined by the loaded $Q$ of the circuit, $Q_L$, which is given by

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{e/\text{left}}} + \frac{1}{Q_{e/\text{right}}} \cdot \tag{8}$$

Here $Q_0$ is the unloaded $Q$ of the resonator element, while the external $Q$'s determine the amount of coupling between the resonator and the supply and load. In terms of the equivalent circuit of Fig. 5(c), these $Q$'s are defined as

$$Q_0 = \frac{\omega L}{r}; \qquad Q_{e/\text{left}} = \frac{\omega L}{Z_0}; \qquad Q_{e/\text{right}} = \frac{\omega L}{R_L} \cdot \tag{9}$$

Narrow bandwidth requires a high value of $Q_L$; while low insertion loss requires that $Q_0$ is much larger than either $Q_{e/\text{left}}$ or $Q_{e/\text{right}}$. For maximum power transfer, the two external $Q$'s should equal each other, although this requirement is not critical.

To compute these $Q$'s, we assume a given amount of energy stored in the plasma ($LC$). This determines the constant $A_1$ of (7), and, therefore, the amplitudes of electric fields. Corresponding to these, we can find the average power lost in the plasma and dielectric to get $Q_0$; the power radiated by the plasma into the waveguide to get $Q_{e/\text{left}}$; and, the power sent into the coupling probe to the load to get $Q_{e/\text{right}}$.

All three calculations require a knowledge of the energy stored at a given field amplitude $A_1$. This is found by integration of the energy density $\frac{1}{2}\epsilon_0 \boldsymbol{E}\cdot\boldsymbol{E}$ in the electric field at the instant when there is no motion of plasma electrons. Although this integration should be carried out over the volume of the waveguide, the approximation of the plasma column in free space is believed to be sufficiently accurate. The relative dielectric constant of the plasma in this computation must be taken as unity instead of the value given by (1).

The result, for waveguide width $a$ and plasma tube inner radius $r_0$ at plasma resonance is

$$U = \left(\frac{1}{4}\right)\epsilon_0 A_1^2 \pi r_0^2 a \left(\frac{\omega_p^2}{\omega_r^2}\right). \tag{10}$$

*A. $Q_0$*

To compute the average power lost in the column, we integrate the specific power loss 1/2 Re $(\sigma)\boldsymbol{E}\cdot\boldsymbol{E}$. Here $\sigma$ is the conductivity, given approximately by

$$\sigma \simeq \epsilon_0 \nu_c \frac{\omega_p^2}{\omega^2} - j\epsilon_0 \frac{\omega_p^2}{\omega}\cdot \tag{11}$$

The result at resonance is

$$P_p = \frac{1}{4}\epsilon_0 \nu_c A_1^2 \pi r_0^2 a \left(\frac{\omega_p^2}{\omega_r^2}\right). \tag{12}$$

The internal $Q$, $Q_0$, is, therefore, simply

$$Q_0 = \frac{\omega_r U}{P_p} = \frac{\omega_r}{\nu_c}\cdot \tag{13}$$

This is the $Q$ of the plasma only. We must also compute the power lost in the dielectric tube. We have found that for our structure at $S$ band the ratio of power lost in the dielectric, $P_d$, to that lost in the plasma is given approximately by

$$\frac{P_d}{P_p} \simeq 0.04 D\epsilon_r Q_0, \tag{14}$$

where $D$ and $\epsilon_r$ are the dissipation factor and relative dielectric constant of the dielectric, respectively.

To keep from lowering the effective internal $Q$, the dielectric tube must have appreciably lower loss than the plasma. For the case given by (14), this means $(D\epsilon_r)$ should be less than about $5/Q_0$. In the following, we will assume that a good dielectric is used, so that the internal $Q$ is essentially $Q_0$.

*B. $Q_{e/left}$*

Since the column behaves as a line of electric dipoles, the relations for an electric dipole radiating into a waveguide can be used.[15] For a oscillating dipole of electric moment $M = ql$, which is located in a rectangular waveguide as in Fig. 6, the radiated electric field in the waveguide dominant $TE_{10}$ mode is

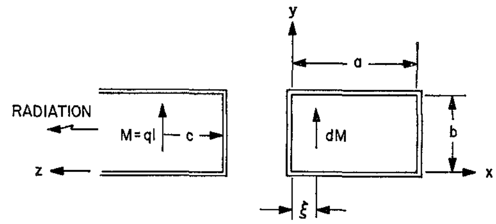


Fig. 6—Radiation of an electric dipole into rectangular waveguide with one end closed.

$$E_y = \frac{2\omega M}{ab}\left(\frac{\mu}{\epsilon}\right)^{\frac{1}{2}} \frac{\sin\frac{\pi\xi}{a}}{\left(1-\left(\frac{\pi}{ka}\right)^2\right)^{1/2}} \sin\frac{\pi x}{a}\sin\beta c\epsilon^{-j\beta Z}, \tag{15}$$

where

$\omega$ = frequency of the oscillating dipole

$a, b$ = waveguide dimensions

$\mu, \epsilon$ = constants of material filling the guide

$k = \omega\sqrt{\mu\epsilon}$

$\beta = k\sqrt{1-(\pi/ka)^2}$

$c$ = distance between dipole and closed end of waveguide.

Considering our plasma column to be composed of a distribution of infinitesimal dipoles of amplitude, $dM = M_0 \sin(\pi\xi/a)d\xi$, across the guide, the total radiated $E_y$ is found by integration to be

$$E_y = \frac{-\omega M_0}{b}\sqrt{\frac{\mu}{\epsilon}} \frac{\sin\beta c}{\sqrt{1-\left(\frac{\pi}{ka}\right)^2}} \sin\frac{\pi x}{a}\epsilon^{-j\beta Z}. \tag{16}$$

The power radiated by the dipole distribution is then

$$P_l = \frac{\omega^2 M_0^2 a}{4b} Z_0[\sin\beta c]^2, \tag{17}$$

where

$$Z_0 = \text{waveguide impedance}$$

$$= \sqrt{\frac{\mu}{\epsilon}} \frac{1}{\left(1-\left(\frac{\pi}{ka}\right)^2\right)^{1/2}}\cdot$$

To compute $Q_{e/\text{left}}$, it remains to relate $M_0$ to $A_1$. We consider a slice of thickness $d\xi$ of the oscillating column, as in Fig. 7. At the peak of a cycle, a charge is found on the surface of the disk, as shown. The total dipole moment of the disk here is

$$dM = (4d\xi)\int_0^r \frac{\rho\, dr}{\cos\phi}\sqrt{r_0^2 - r^2}, \tag{18}$$

[15] J. C. Slater, "Microwave Transmission," McGraw-Hill Book Co., Inc., New York, N. Y.; 1942.

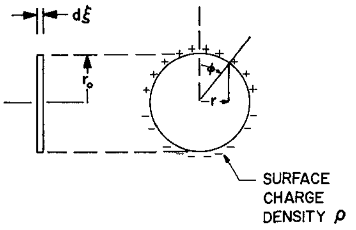


Fig. 7—Dipole moment of section of oscillating column.

where

$\rho$ = surface charge density,
$r, \phi$ = polar coordinates,
$r_0$ = radius of the column.

The surface-charge density is the difference in normal, or radial electric flux density, when the dielectric constant of the plasma is taken as unity.

$$\rho = \left[\epsilon_0(E_{1r} - \epsilon_2 E_{2r})\,|_{r=r_0}\right]\left[\sin\frac{\pi\xi}{a}\right]. \quad (17)$$

Here $E_{1r}$, $E_{2r}$ are radial electric fields in plasma and dielectric, respectively.

From (6) and (7), $\rho$ can be found in terms of the dimensions, the dielectric constant $\epsilon_2$, and the amplitude $A_1$. Alternately, by expressing $E_{1r}$ and $E_{2r}$ in terms of $\epsilon_p$ and $\epsilon_2$, we arrive at the relation

$$\rho = \epsilon_0(1 - \epsilon_p)A_1 \cos\phi \sin\frac{\pi\xi}{a}, \quad (18)$$

where $\epsilon_p$ is given by (1) at the resonant frequency (2).

From the integration of (18) and the definition of $dM$, we now get

$$M_0 = \pi r_0^2\epsilon_0(1 - \epsilon_p)A_1 = \pi r_0^2\epsilon_0 A_1\left(\frac{\omega_p^2}{\omega_r^2}\right). \quad (19)$$

From (17) and (10), we finally get

$$Q_{e/\text{left}} = \frac{\omega_r U}{P_l}\cdot\frac{b}{\omega_r\pi r_0^2\epsilon_0\left(\frac{\omega_p^2}{\omega_r^2}\right)Z_0\sin\beta c}. \quad (20)$$

As expected, the coupling is decreased when the relative amount of guide height filled by the column is decreased, when the number of radiating electrons is decreased ($\omega_r \propto N^{1/2}$), or when the impedance seen by the column is decreased by a change in distance $c$.

For the case investigated experimentally, a numerical calculation results in the minimum value of $Q_{e/\text{left}}$ of about 4. It is seen, therefore, that the column can be easily overcoupled on the input side.

### C. $Q_{e/right}$

While the calculation of $Q_{e/\text{left}}$ is believed to be quite accurate for a column of uniform plasma density, we have resorted to an approximation to estimate $Q_{e/\text{right}}$.

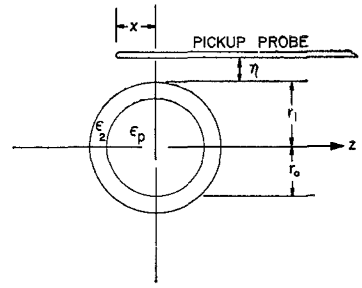


Fig. 8—Probe coupling to plasma column.

Shown in Fig. 8 is the probe, column, and an appropriate coordinate system.

The electric field outside the column was shown to be

$$E_3 = [\hat{\imath}_r\cos\phi + \hat{\imath}_\phi\sin\phi]\left[-A_3 r^{-2}\sin\frac{\pi x}{a}\right]. \quad (21)$$

We now assume that the voltage induced on the probe is the line integral of the parallel component of this electric field along it.

$$V_{\text{probe}} \simeq \int_{-x}^{\infty} E_{3z}\,|_{y=r_1+\eta}\,dz = \frac{A_3(r_1+\eta)}{(r_1+\eta)^2 + \chi^2}. \quad (22)$$

If the probe feeds a coaxial line of impedance $Z_{oc}$, the power coupled out becomes

$$P_r = \frac{1}{2}\frac{V_{\text{probe}}^2}{Z_{oc}} = \frac{1}{2}\frac{A_3^2(r_1+\eta)^2}{[(r_1+\eta)^2+\chi^2]^2}\cdot\frac{1}{Z_{oc}}. \quad (23)$$

By expressing $A_3$ in terms of $A_1$, we find from (7) and (10),

$$Q_{e/\text{right}} = \frac{2\epsilon_0\cdot\left(\frac{\omega_p^2}{\omega_2}\right)\epsilon_2^2\pi a\left(\frac{r_0}{r_1}\right)^2 Z_{oc}\omega}{\left[\epsilon_p\left(1-\frac{r_0^2}{r_1^2}\right)+\epsilon_2\left(1+\frac{r_0^2}{r_1^2}\right)\right]^2}\cdot\left[\frac{\left(1+\frac{\eta}{r_1}\right)^2+\frac{\chi^2}{r_1^2}}{1+\frac{\eta}{r_1}}\right]^2. \quad (24)$$

The coupling here is now obviously decreased by retracting the probe from the column, and also by widening the waveguide. Here, again, a numerical calculation results in the minimum value of $Q_{e/\text{right}}$ of about 4. Since the probe can be retracted as much as desired, the column can be easily overcoupled on the output side.

## IV. Experimental Data

Two types of waveguide filters were built and tested at $S$ band using standard RG-48/U guide. In one filter, the output probe was used to feed another waveguide, as shown in Fig. 9; in the second, shown in Fig. 10, the probe fed a coaxial line. In both cases, the filters behaved in the manner predicted, although the results

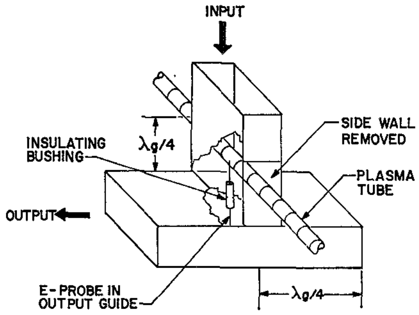


Fig. 9—Plasma filter with waveguide output (exposed view)

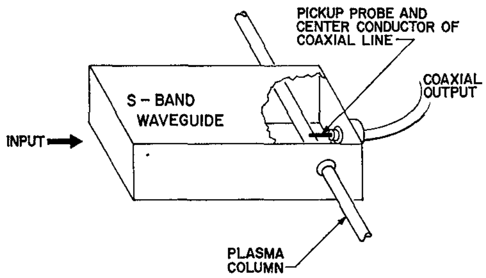


Fig. 10—Plasma filter with coaxial output (exposed view).

were not quite in numerical agreement with the calculations of the $Q_e$'s.

The plasma tubes were positive columns of discharge tubes that had a thyratron-type hot cathode, a mercury pool reservoir, and a starting anode close to the cathode. The mercury gas pressure was estimated from temperature measurements to be on the order of 0.1 mm Hg. The discharge was maintained by a dc-voltage supply in series with ballast resistors and the tube. Normal operating conditions were about 50-volts drop across the tube, for about 0.4 amperes discharge current. The positive column portion of a discharge tube was of 6 mm ID, 8 mm OD, and approximately 45 cm long. Both quartz and pyrex were used. The latter was preferred in the experiments, since it did not transmit the ultraviolet radiations of the discharge. It was computed that the higher dielectric losses in pyrex should not appreciably affect the operation of the prototype tubes.

While these tubes were quite satisfactory for demonstrating the operation of the filter, it became obvious very quickly that considerable improvements in the discharge tube would be needed before a plasma filter could be considered a reliable piece of equipment. The chief difficulties found were that the plasma density was very temperature dependent, that it drifted occasionally, and the column tended to oscillate at frequencies from the kilocycle to the megacycle range. Following the work of Crawford,[16] we were usually able to reduce these oscillations considerably by a small magnetic field in the vicinity of the cathode.

[16] F. W. Crawford and G. S. Kino, "Oscillations and noise in low-pressure dc discharges," Proc. IRE, vol. 49, pp. 1767–1788; December, 1961.

The plasma density could easily be varied in a controlled manner by varying the discharge current. Both a 400-cps sine wave and a 3000-cps sawtooth were used, at different times, for sweeping the current. While mercury limits the rate of tunability to frequencies of the kilocycle range, the use of the lighter gases should increase the possible speed of operation considerably.

A typical CRO trace demonstrating the operation of the filter is shown in Fig. 11, which presents detected output power vs plasma current for the waveguide out put filter. Here the short in the input waveguide was placed $\lambda_g/4$ (at the center frequency) behind the discharge tube. The coupling probe was 0.5 mm diameter, located in the center of the guides, and extended slightly beyond the plasma tube in the input guide. The signal frequency was fixed at 3450 Mc; the plasma current varied from 0.1 to 0.9 amperes.

It is seen on Fig. 11 that output power exists only when the plasma density corresponds to near the resonant value. Fig. 11 can be considered to demonstrate the principle of operation of the plasma filter.

Typical frequency response curves of the waveguide output filter at fixed plasma densities are shown in Fig. 12. As the discharge current and, hence, the plasma density is increased, the pass band moves to higher frequencies, as predicted.

For the discharge voltage adjusted to place the dipole resonance at various frequencies throughout the *S*-band range, the 3-db bandwidth of the waveguide output filter was measured. The results are shown in Table I. A typical 3-db bandwidth was 150 to 200 Mc, corresponding to a loaded $Q$ of about 15 to 20. At the same time, the insertion loss was less than 2 db over most of the range. A curve of insertion loss is shown in Fig. 13. Outside of the range listed in Table 1, the insertion loss increased, possibly because of the change in the $Q_e$'s as the distance between plasma tube and input waveguide short differed appreciably from $\lambda_g/4$; and, as the effective coupling to the output guide changed.

The filter of Fig. 10 performed in about the same manner. A typical $Q_L$ at 3500 Mc was 19, with an insertion loss of 1.6 db. Here it was found that the best performance was obtained with input coupling reduced by a decrease in distance between plasma tube and waveguide from that of midband $\lambda_g/4$.

While the performance of the filter has been demonstrated, there are several points that remain to be discussed. In particular, our calculations for the configuration used predict a loaded $Q$ of about 2, while actually we measured about 20. Furthermore, the important quantity of interest is $Q_0$, for it chiefly determines the how narrow a bandwidth is attainable.

### *A. Measurement of $Q_0$*

The unloaded $Q$ of the column was measured by decoupling both input and output circuit to a high degree, then measuring the output power as the input frequency was swept through the pass band. This was ac-

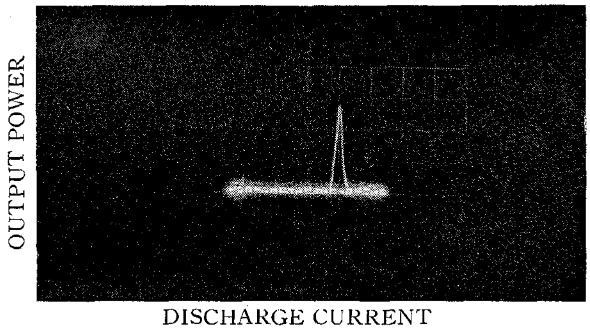


Fig. 11—Oscilloscope trace of output power vs discharge current at fixed frequency.

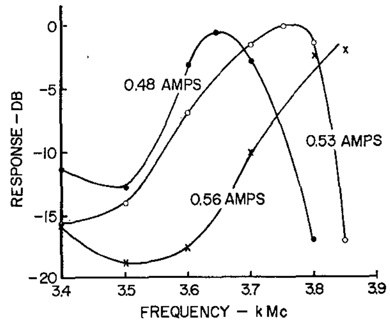


Fig. 12—Response of waveguide output filter vs frequency at 3 values of discharge current.

TABLE I

Bandwidth of Plasma Filter at S Band

| Center Frequency | 3-db Bandwidth | Discharge Current | Insertion Loss |
|---|---|---|---|
| 3295 Mc | 320 Mc | 0.5 ampere | |
| 3400 Mc | 230 Mc | 0.51 ampere | |
| 3499 Mc | 140 Mc | 0.52 ampere | $<2$ db over entire range |
| 3589 Mc | 170 Mc | 0.57 ampere | |
| 3695 Mc | 125 Mc | 0.64 ampere | |
| 3789 Mc | 150 Mc | 0.68 ampere | |
| 3891 Mc | 130 Mc | 0.70 ampere | |
| 3982 Mc | 80 Mc | 0.77 ampere | |

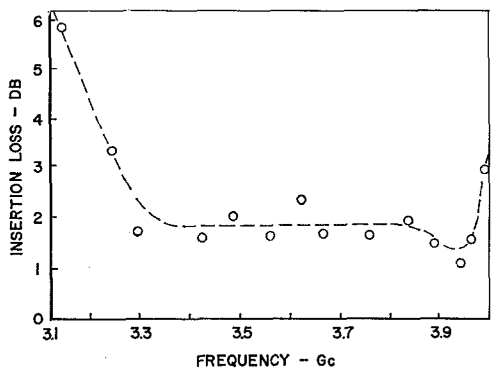


Fig. 13—Insertion loss at center frequency vs center frequency (for the waveguide output filter).

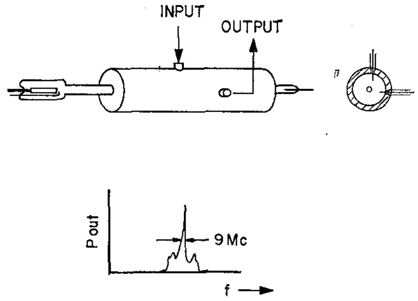


Fig. 14—Measurement of $Q_0$. (a) Physical configuration. (b) Output waveform.

complished by placing the column in a small-diameter metal tube, as shown in Fig. 14, into which two probes connected to coaxial lines were inserted a short distance. The probes were separated from each other 7.5 cm and were geometrically at right angles. The resulting output signal had a 3-db bandwidth of 9 Mc, corresponding to $Q_0$ of 390.

This measurement also brought out another property of the plasma column used. In addition to the main peak of coupled power, there were also smaller peaks at both sides of this main resonance not generally seen in the other filter experiments. It is believed that these were due to the nonuniformity of the plasma density throughout the column. Such smaller peaks would not be as highly excited by the symmetrical fields of the $TE_{10}$ waveguide as with the double probe coupling.

It was also thought that the configuration of Fig. 14 itself could be used as a filter. However, while this filter will pass the frequency of plasma resonance, it also couples through energy of this frequency when the plasma density is raised to the value necessary for the propagation of the space-charge waves discussed by Trivelpiece and Gould.[17] This high-density propagation was observed here.

*B. External Q's*

The computation of external $Q$'s was based on the idealized case of a plasma column of uniform density. Actually, the density is expected to vary both radially and axially. In particular, if the density varies appreciably on both sides of the coupling probe, only a fraction of the column is active in the filter. This effect reduces the coupling to the waveguide, thereby increasing $Q_{e/\text{left}}$. The value of $Q_L=19$ found, therefore, follows this observation. Furthermore, it is not too surprising that $Q_{e/\text{right}}$ is different from the value computed, not only because of the probable variation of radial density, but also because the method used in the computation here was recognized to be an approximate one only.

[17] A. W. Trivelpiece and R. W. Gould, "Space charge waves in cylindrical plasma oclumns," *J. Appl. Phys.*, vol. 30, pp. 1784–1793; November, 1959.

The point to be emphasized is that, as predicted, sufficiently low values of the $Q_e$'s were found to exist in the experimental work for the construction of a filter of less than 2-db insertion loss with $Q_0=390$. If it should be possible to construct plasma tubes of higher $Q_0$, we can be reasonably assured that filters of narrower percentage bandwidths than our experimental prototype can be constructed, for the external $Q$'s can always be raised by decreasing the coupling.

### C. Reciprocity

The action of the plasma filter is reciprocal. This has been demonstrated in the laboratory.

### D. Higher-Order Modes

For a plasma column of finite length, (4) and (5) state that a number of resonant frequencies should exist. For a long column, the lower orders coalesce to a single resonance; for a short column, the resonances with $m>1$ should appear on the high-density side of the dominant resonance. Such modes were seen in the laboratory when the holes through which the plasma tube was inserted into the waveguide were only slightly larger than the tube. The effect was enhanced when the active length of the tube was further reduced by metal blocks inserted into the guide.

These higher-order modes were minimized when the holes were greatly enlarged as in Fig. 11; or, when the waveguide sides were effectively extended by metal tubes concentric with, and of considerably larger diameter than, the discharge tube.

## V. Conclusions

The experimental results verify that the plasma filter behaves in general accordance with the simplified theory presented. The dipolar plasma-column resonance can be treated as a typical microwave resonator. It has been demonstrated that efficient coupling over a considerable portion of a waveguide band is possible.

In the prototype filter tested, the loaded $Q$ possible has not been as high as in perturbed cavity or single crystal YIG filters. This is due to the lower unloaded $Q$ of the mercury plasma, which is limited by the rather high collison frequency. It may be possible to increase the unloaded $Q$ considerably by the use of gases of lower collision frequency. This possiblity was not explored in these initial experiments.

The question of effective collision frequency, as applied to the plasma filter, is still being debated in some circles. In particular, the effect of wall collisions on the microwave conductivity is not completely settled. If these wall collisions play the dominant role in determining the $Q$ of the column, then not too great an improvement $Q$ can be expected by the use of different gases unless the electron temperature can also be reduced. Experiments for testing the effect of these wall collisions on the plasma $Q$ appear to be in order.

The possible tuning speed of the plasma filter is determined chiefly by the de-ionization time of the plasma. For typical gases, this de-ionization time is from hundreds of microseconds to a few microseconds. The filter is, therefore, rapidly tunable.

No measurements of the noise properties of the filter were made. While the electron temperature of a positive column plasma is generally quite high, only a small fraction of the noise originating from these electrons will be radiated into the filter if the insertion loss is kept low.

As mentioned earlier in this paper, while the experimental tubes used demonstrated the principle, they are not yet to be considered as reliable microwave components. Considerable improvements are required, chiefly to reduce instabilities and low-frequency oscillations.

If these improvements can be achieved, the plasma filter could well find applications in systems in which rapid electronic tuning is required.

## Acknowledgment

It is a pleasure to acknowledge the work of R. P. Kemp in the laboratory.